



# 個體經濟學一

Microeconomics (I)

## CH 5 More on the analysis of consumer behavior

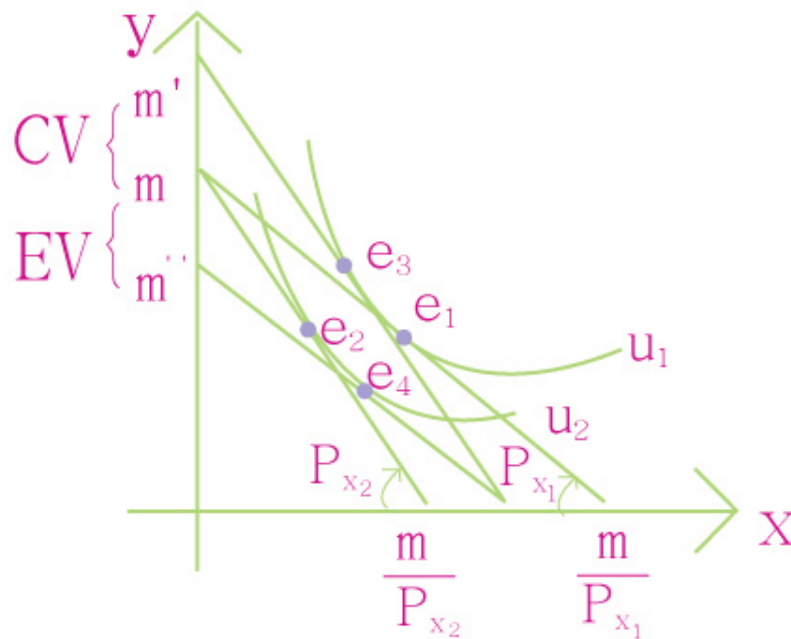


Figure 74 :

An increase in the price of  $X$ ,  $P_x \uparrow$

$$P_{x_1} \rightarrow P_{x_2}, \quad P_{x_2} > P_{x_1}$$

Assume  $P_y = 1$  and  $m$  are fixed.

$$m' = e(P_{x_2}, P_y, u_1) \quad m = e(P_{x_1}, P_y, u_1) = e(P_{x_2}, P_y, u_2)$$

$$m'' = e(P_{x_1}, P_y, u_2)$$

$$CV = e(P_{x_2}, P_y, u_1) - m = e(P_{x_2}, P_y, u_1) - e(P_{x_2}, P_y, u_2)$$

distance between  $u_1$  and  $u_2$  in terms of new prices

$$EV = e(P_{x_1}, P_y, u_2) - m = e(P_{x_1}, P_y, u_2) - e(P_{x_1}, P_y, u_1)$$

distance between  $u_2$  and  $u_1$  in terms of old prices

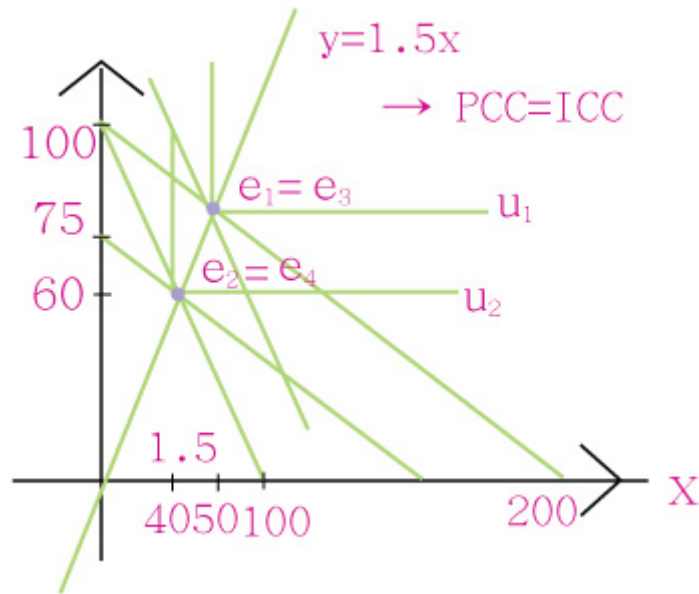


Figure 70 :

EX:  $X$  &  $Y$  are perfect complement.

$$u(x,y) = \min\left\{\frac{x}{2}, \frac{y}{3}\right\}$$

$$(P_{x_1}, P_y, m) = (0.5, 1, 100)$$

$$(P_{x_2}, P_y, m) = (1, 1, 100)$$

$$\text{most efficient } x,y \text{ ratio: } \frac{x}{2} = \frac{y}{3} \Rightarrow y = 1.5x$$

$$e_1 \text{ satisfies } \begin{cases} y = 1.5x \\ 0.5x + y = 100 \end{cases} \Rightarrow \begin{cases} x = 50 \\ y = 75 \end{cases}$$

$$u(x,y) = \min\left\{\frac{x}{2}, \frac{y}{3}\right\} \Rightarrow u_1 = 25$$

$$e_2 \text{ satisfies } \begin{cases} y = 1.5x \\ x + y = 100 \end{cases} \Rightarrow \begin{cases} x = 40 \\ y = 60 \end{cases}$$

$$u(x,y) = \min\left\{\frac{x}{2}, \frac{y}{3}\right\} \Rightarrow u_2 = 20$$

$$\text{since } e_3 = e_1 = (50, 75) \Rightarrow m' = 1 \cdot 50 + 1 \cdot 75 = 125$$

$$CV = 125 - 100 = 25$$

$$\text{since } e_4 = e_2 = (40, 60) \Rightarrow m'' = 0.5 \cdot 40 + 1 \cdot 60 = 80$$

$$EV = 80 - 100 = -20$$

➤ **ordinary demand functions**

$$\max_{x,y} \left( \min\left\{\frac{x}{2}, \frac{y}{3}\right\} \right)$$

$$\text{s.t. } P_x x + P_y y = m$$

$$\begin{cases} \frac{x}{2} = \frac{y}{3} \\ P_x x + P_y y = m \end{cases} \Rightarrow \begin{cases} y = 1.5x \\ P_x x + P_y y = m \end{cases}$$

$$P_x x + 1.5x P_y = m$$

$$(P_x + 1.5P_y)x = m$$

$$x^* = \frac{m}{P_x + 1.5P_y} = \frac{2m}{2P_x + 3P_y}$$

$$y^* = \frac{1.5m}{P_x + 1.5P_y} = \frac{3m}{2P_x + 3P_y}$$

➤ **Indirect utility function**

$$V(P_x, P_y, m) = u(x^*, y^*) = \frac{m}{2P_x + 3P_y}$$

➤ **Compensated demand functions**

$$\min_{x,y} P_x x + P_y y$$

$$\text{s.t. } \min\left\{\frac{x}{2}, \frac{y}{3}\right\} = u$$

$$\left. \begin{aligned} \frac{x}{2} = \frac{y}{3} = u &\Rightarrow x^h = 2u \\ &y^h = 3u \end{aligned} \right\} \text{ independent of } P_x, P_y$$

➤ **Expenditure function**

$$e(P_x, P_y, u) = P_x x^h + P_y y^h = P_x \cdot 2u + P_y \cdot 3u = (2P_x + 3P_y)u$$

$$u_1 = V(P_{x_1}, P_y, m) = V(0.5, 1, 100) = \frac{100}{2 \cdot 0.5 + 3 \cdot 1} = 25$$

$$u_2 = V(P_{x_2}, P_y, m) = V(1, 1, 100) = \frac{100}{2 \cdot 1 + 3 \cdot 1} = 20$$

$$CV = e(P_{x_2}, P_y, u_1) - m = e(1, 1, 25) - 100 = (2 \cdot 1 + 3 \cdot 1) \cdot 25 - 100 = 25$$

$$EV = e(P_{x_1}, P_y, u_2) - m = e(0.5, 1, 20) - 100 = (2 \cdot 0.5 + 3 \cdot 1) \cdot 20 - 100 = -20$$

**EX:**

$$u(x,y)=x^2y$$

$$(P_{x_1}, P_y, m)=(0.5,1,90)$$

$$(P_{x_2}, P_y, m)=(1,1,90)$$

➤ **ordinary demand functions**

$$\max_{x,y} x^2y$$

$$\text{s.t. } P_x x + P_y y = m$$

$$x^* = \frac{2}{2+1} \frac{m}{P_x} = \frac{2}{3} \frac{m}{P_x}$$

$$y^* = \frac{1}{2+1} \frac{m}{P_y} = \frac{1}{3} \frac{m}{P_y}$$

➤ **Indirect utility function**

$$V(P_x, P_y, m) = x^{*2} y^* = \left(\frac{2}{3} \frac{m}{P_x}\right)^2 \left(\frac{1}{3} \frac{m}{P_y}\right)$$

$$u_1 = V(0.5, 1, 90) = 432000 \quad x_1 = 120, y_1 = 30$$

$$u_2 = V(1, 1, 90) = 108000 \quad x_2 = 60, y_2 = 30$$

➤ **Compensated demand functions**

$$\min_{x,y} P_x x + P_y y$$

$$\text{s.t. } x^2 y = u$$

$$\text{Foc } MRS_{xy} = \frac{P_x}{P_y} \quad \Phi$$

$$x^2 y = u \quad \Theta$$

$$\text{LHS of } \Phi \quad MRS_{xy} = \frac{Mu_x}{Mu_y} = \frac{2xy}{x^2} = \frac{2y}{x}$$

$$\Phi \Rightarrow \frac{2y}{x} = \frac{P_x}{P_y} \Rightarrow y = \frac{P_x x}{2P_y}$$

$$\Theta \Rightarrow x^2 \frac{P_x}{2P_y} x = u$$

$$\frac{P_x}{2P_y} x^3 = u$$

$$x^3 = \frac{2P_y}{P_x} u$$

$$x^h = \sqrt[3]{\frac{2P_y}{P_x} u}$$

$$y^h = \frac{P_x}{2P_y} \sqrt[3]{\frac{2P_y}{P_x} u} = \sqrt[3]{\frac{P_x^2}{4P_y^2} u}$$

➤ **Expenditure function**

$$e(P_{X_1}, P_Y, u_2) = P_X x + P_Y y = P_X \sqrt[3]{\frac{2P_Y}{P_X} u} + P_Y \sqrt[3]{\frac{P_X^2}{4P_Y^2} u} = \sqrt[3]{2P_X^2 P_Y u} + \sqrt[3]{0.25P_X^2 P_Y u}$$

$$\begin{aligned} e(P_{X_2}, P_Y, u_1) &= e(1, 1, 432000) = \sqrt[3]{864000} + \sqrt[3]{108000} \\ &= \sqrt[3]{2^3 * 108000} + \sqrt[3]{108000} \\ &= 3\sqrt[3]{108000} = 90\sqrt[3]{4} \end{aligned}$$

$$\begin{aligned} e(P_{X_1}, P_Y, u_2) &= e(0.5, 1, 108000) = \sqrt[3]{0.5 * 108000} + \sqrt[3]{\frac{1}{16} 108000} \\ &= 30\sqrt[3]{2} + 15\sqrt[3]{2} = 45\sqrt[3]{2} \end{aligned}$$

$$CV = 90\sqrt[3]{4} - 90$$

$$EV = 45\sqrt[3]{2} - 90$$

$(P_{X_1}, P_Y, m) \rightarrow x^*, y^*$  old equilibrium

$$V(P_{X_1}, P_Y, m) = u(x^*, y^*) = u_1$$

$(P_{X_2}, P_Y, m) \rightarrow x', y'$  new equilibrium

$$V(P_{X_2}, P_Y, m) = u(x', y') = u_2$$

$e(P_X, P_Y, u)$  is the expenditure function

$\min_{x,y} P_X x + P_Y y$   $(x^h, y^h)$  compensated demand function

$$\text{s.t. } u(x,y) = u \quad x^h = x(P_X, P_Y, u)$$

$$y^h = y(P_X, P_Y, u)$$

$$e(P_X, P_Y, u) = P_X x^h + P_Y y^h$$

$$e(P_{X_1}, P_Y, u_2) = e(P_{X_1}, P_Y, u_1) = m$$

$$e(P_{X_2}, P_Y, u_1) = e(P_{X_2}, P_Y, u_2) = m$$

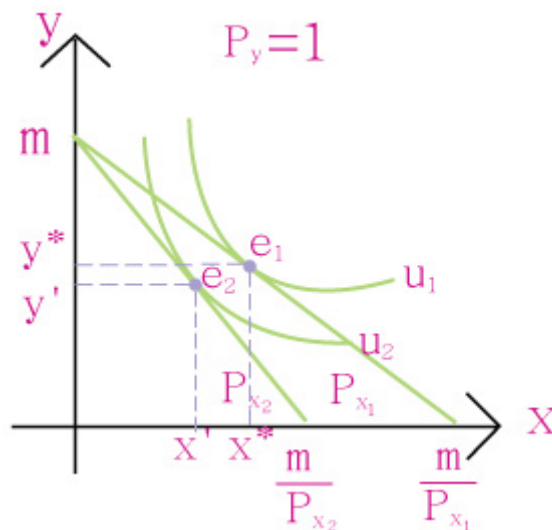


Figure 71 :

**結論 :**  $CV = e(P_{X_2}, P_Y, u_1) - m = e(P_{X_2}, P_Y, u_1) - e(P_{X_2}, P_Y, u_2)$

$EV = e(P_{X_1}, P_Y, u_2) - m = e(P_{X_1}, P_Y, u_2) - e(P_{X_1}, P_Y, u_1)$

EX:

$$u(x,y) = \sqrt{x} + y$$

$$\max_{x,y} \sqrt{x} + y$$

$$\text{s.t. } P_x x + P_y y = m$$

$$\left. \begin{array}{l} \text{Foc } \text{MRS}_{xy} = \frac{P_x}{P_y} \quad \phi \\ P_x x + P_y y = m \quad \phi \end{array} \right\} x^*, y^*$$

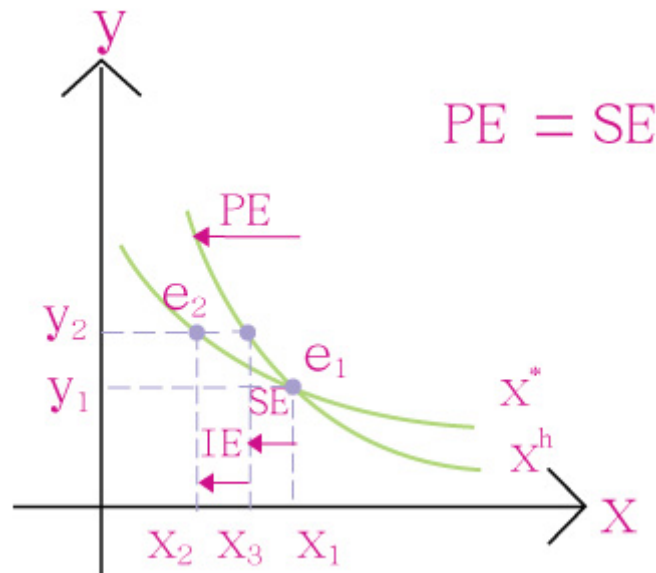
$$\phi \text{ LHS } \text{MRS}_{xy} = \frac{M u_x}{M u_y} = \frac{\frac{1}{2} x^{-\frac{1}{2}}}{1} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2x^{0.5}}$$

$$\phi \rightarrow \frac{1}{2x^{0.5}} = \frac{P_x}{P_y} \rightarrow x^* = \frac{P_y^2}{4P_x^2}$$

note that in the process of finding compensating demand function, we have the FOC:

$$\left. \begin{array}{l} \text{MRS}_{xy} = \frac{P_x}{P_y} \quad \phi \\ \sqrt{x} + y = u \quad \phi' \end{array} \right\} \Rightarrow \begin{array}{l} x^h \\ y^h \end{array}$$

$$\phi \Rightarrow x^h = \frac{P_y^2}{4P_x^2} \quad x^* = x^h$$



$$PE = SE + IE$$

In this example,  $x_2 = x_3$   
 $\Rightarrow PE = SE$ , there is no income effect.

Figure 72 :

$$\emptyset \Rightarrow P_x \cdot \frac{P_y^2}{4P_x^2} + P_y y = m$$

$$P_y y = m - \frac{P_y^2}{4P_x^2}$$

$$y^* = \frac{m}{P_y} - \frac{P_y}{4P_x}$$

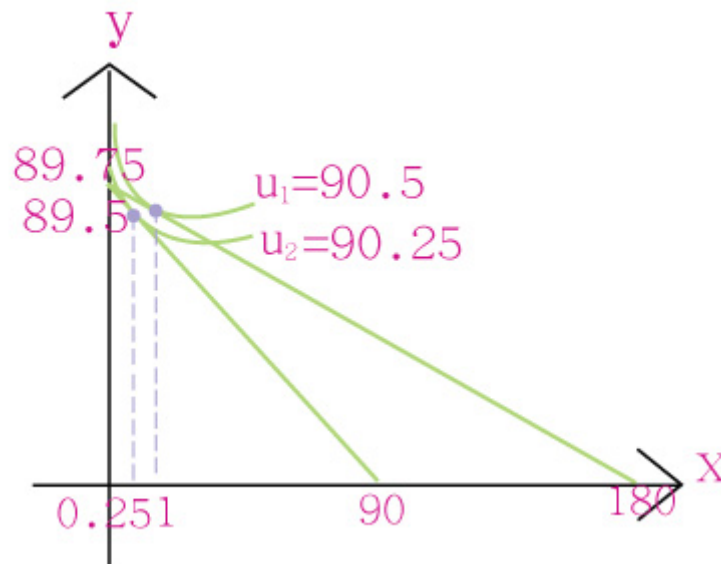


Figure73 :

$$P_{x1} = 0.5, P_y = 1, m = 90$$

$$P_{x2} = 1, P_y = 1, m = 90$$

$$V(P_x, P_y, m) = \sqrt{\frac{P_y^2}{4P_x^2}} + \frac{m}{P_y} - \frac{P_y}{4P_x}$$

$$= \frac{P_y}{2P_x} + \frac{m}{P_y} - \frac{P_y}{4P_x}$$

$$= \frac{m}{P_y} + \frac{P_y}{4P_x}$$

$$u_1 = V(P_{x1}, P_y, m) = V(0.5, 1, 90) = 90.5$$

$$x^* = 1, y^* = 89.5$$

$$u_2 = V(P_{x2}, P_y, m) = V(1, 1, 90) = 90.25$$

$$x' = 0.25, y' = 89.75$$

$$e(P_x, P_y, u) = ?$$

$$\min_{x,y} P_x x + P_y y$$

$$\text{s.t. } \sqrt{x} + y = u$$

$$\text{FOC. } \left. \begin{array}{l} MRS_{xy} = \frac{P_x}{P_y} \quad \phi \\ \sqrt{x} + y = u \quad \phi' \end{array} \right\} \Rightarrow \begin{array}{l} x^h \\ y^h \end{array}$$

$$\phi \Rightarrow x^h = x^* = \frac{P_y^2}{4P_x^2} \quad u \text{ 不影響}$$

$$\phi' \Rightarrow \frac{P_y}{2P_x} + y = u$$

$$y^h = u - \frac{P_y}{2P_x}$$

$$e(P_x, P_y, u) = P_x \cdot \frac{P_y^2}{4P_x^2} + P_y \left( u - \frac{P_y}{2P_x} \right)$$

$$= \frac{P_y^2}{4P_x} + P_y u - \frac{P_y^2}{2P_x}$$

$$= P_y u - \frac{P_y^2}{4P_x}$$

$$\text{CV} = e(P_{x2}, P_y, u_1) - m$$

$$= e(1, 1, 90.5) - m$$

$$= (90.5 - 0.25) - 90 = 0.25$$

$$\text{EV} = e(P_{x1}, P_y, u_2) - m$$

$$= e(0.5, 1, 90.25) - m$$

$$= (90.25 - 0.5) - 90 = -0.25$$

$$\text{CV} = -\text{EV}$$

### \* quasi-linear function

quasi-linear in y utility function

$$u(x, y) = y + f(x)$$

$$\text{CV} = -\text{EV}$$

### \* Quasi-linear utility function

$$u(x, y) = y + f(x)$$

$$\text{given } u(x, y) = u_1$$

an indifference curve

$$\{(x, y) \mid u(x, y) = u_1\}$$

$$= \{(x, y) \mid y + f(x) = u_1\}$$

$$= \{(x, y) \mid y = u_1 - f(x)\}$$

$$u(x, y) = u_2$$

$$\text{another indifference curve } \{(x, y) \mid y = u_2 - f(x)\}$$



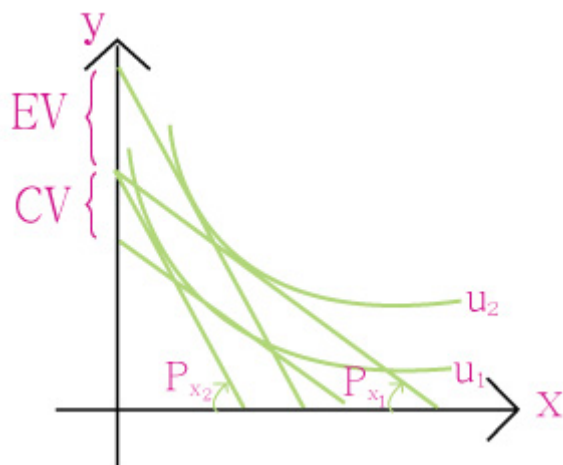


Figure 74 : Equivalent variation (EV) and compensating variation (CV)

the vertical distance between those two difference curve

(suppose  $u_2 > u_1$ )

$y_2 - y_1 = u_2 - u_1$  ( $f(x)$  canceled)

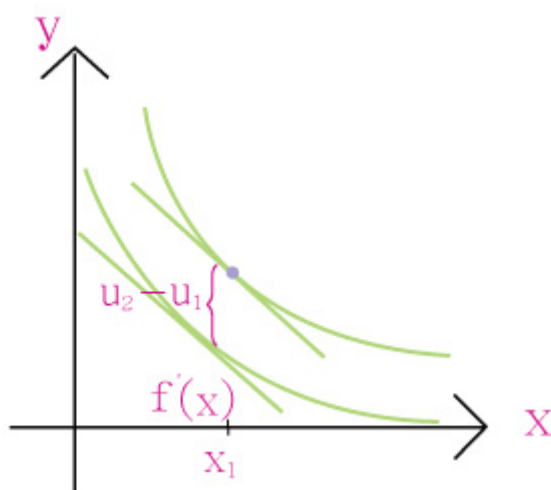


Figure 75 :

given a X

$$MRS_{xy} = \frac{MU_x}{MU_y} = \frac{f'(x)}{1} = f'(x) \quad \text{no } y$$

$\Rightarrow$  given a X,  $f(x) = MRS_{xy}$  is independent of Y.

Since  $MRS_{xy} = f'(x)$

From the FOC,  $MRS_{xy} = \frac{P_x}{P_y}$


$$f'(x) = \frac{P_x}{P_y} \Rightarrow x^* = x^h \text{ is a function of } P_x \text{ and } P_y$$

(no m, no income effect)

$$PE = SE$$

$$IE = 0$$

**\* Consumer Surplus, CS**

A consumer is willing to pay  for  $X_1$  units of  $X$   
units of  $X$

The consumer pays  $P_{X_1} * X_1$  for  $X_1$  units of  $X$

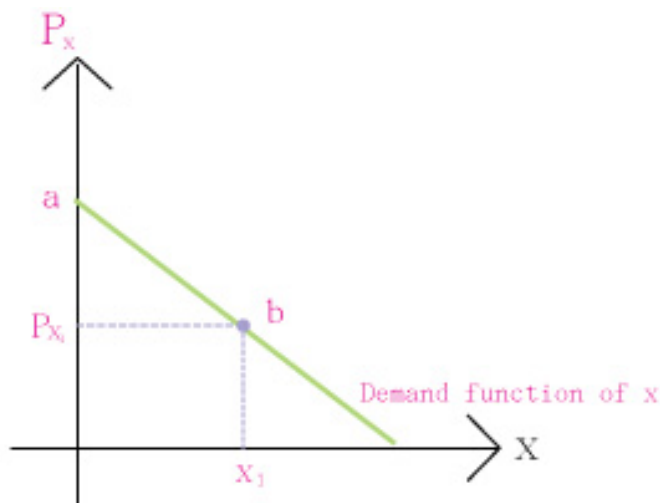
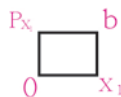
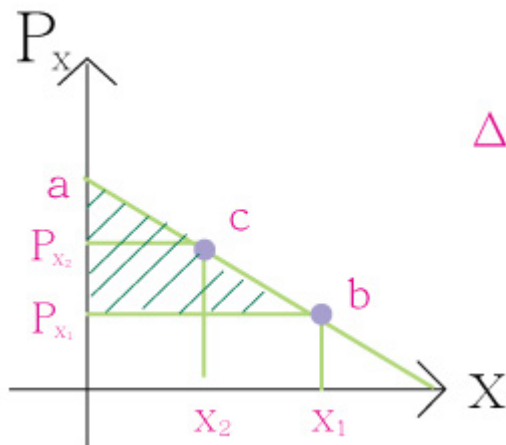


Figure76 :

$$CS = \text{triangle } a-b-x_1 - \text{rectangle } 0-P_x-x_1 = \int_0^{X_1} P_x dx - P_{X_1} X_1$$

an increase of price of  $X$  from  $P_{X_1}$  to  $P_{X_2}$ ,  $P_{X_2} > P_{X_1}$



$$\Delta CS = \text{trapezoid } P_{X_2}-c-b-P_{X_1}$$

a change of the welfare of a consumer

Figure77 :Change in consumer surplus with an increase of price of  $x$

**\* Relationship among CV, EC & ΔCS**

$$a \sim b \sim c \sim d$$

$$(0, y_0) \sim (1, y_1) \sim (2, y_2) \sim (3, y_3)$$

the consumer is willing to pay  $(y_0 - y_1)$

for the 1st unit of X

2nd  $(y_1 - y_2)$

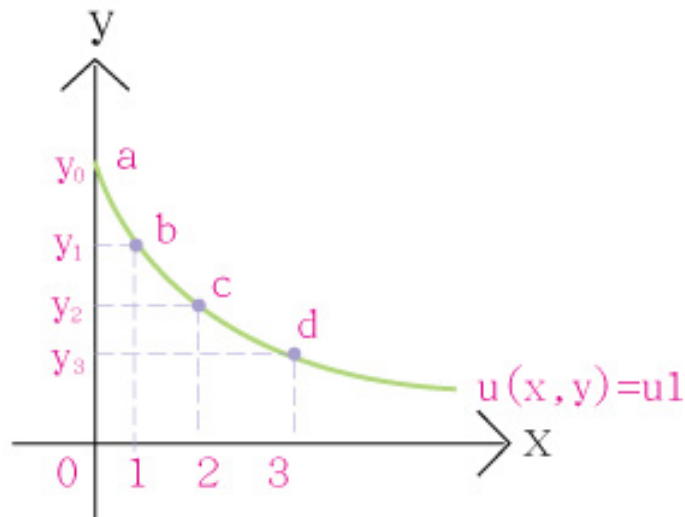


Figure 78 :

$P_y = 1$ ,  $y$  : other expenditure.

$$\text{MRS}_{xy} \text{ at } x=1 = y_0 - y_1$$

$$\text{at } x=2 = y_1 - y_2$$

$$\text{at } x=3 = y_2 - y_3$$

$$\text{in equilibrium, } \text{MRS}_{xy} = \frac{P_x}{P_y}$$

$$P_y = 1 \Rightarrow \text{MRS}_{xy} = P_x$$

(or  $MV_x$ )

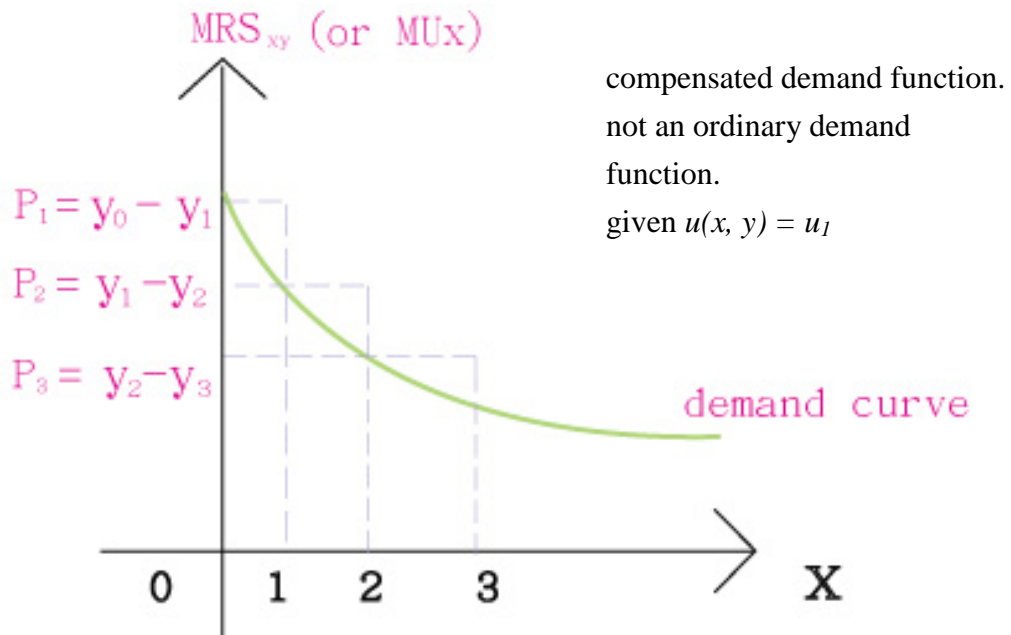


Figure79 :

$$\Delta CS = ?$$

base on  $X^*$

$$\Delta CS \text{ base on } x^h(u_1) = ? \quad CV$$

$$\Delta CS \text{ base on } x^h(u_2) = ? \quad EV$$

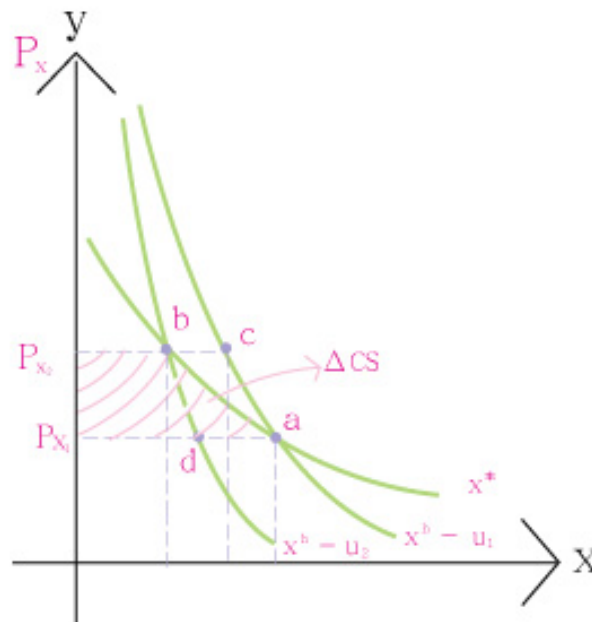


Figure80 :

結論:  $P_{x1} \rightarrow P_{x2}$ ,  $P_{x2} > P_{x1}$

$X$  is normal  $\Rightarrow CV > \Delta CS > EV$

Quasi-linear  $u(x, y) \Rightarrow CV = \Delta CS = EV$

### \* Revealed Preference

The theory of revealed preference.

Bundle  $(x_1, y_1)$  is revealed preferred to bundle  $(x_2, y_2)$

if  $\phi$  both bundles are affordable/

$\phi (x_1, y_1)$  is chosen (but not  $(x_2, y_2)$ )

$$(\phi P_x x_1 + P_y y_1 \geq P_x x_2 + P_y y_2)$$

### \* The principle of the revealed preference.

Suppose a consumer is rational and  $(x_1, y_1)$  is revealed preferred to  $(x_2, y_2)$ , then  $(x_1, y_1)$  must be preferred to  $(x_2, y_2)$

(Axiom)

The weak axiom of revealed preference. (WARP)

Suppose 「 $(x_1, y_1)$  is revealed preferred to  $(x_2, y_2)$ 」 statement A

then 「 $(x_1, y_1)$  cannot be revealed preferred to  $(x_2, y_2)$ 」 statement B

statement A is true  $\Rightarrow$  according to the principle of the revealed preference,  
we have  $(x_1, y_1) > (x_2, y_2)$

statement B is true  $\Rightarrow$

an inconsistency in the consumer's preference.

### EXAMPLE

某人將全部所得用於買 A 物及 B 物。

當 A 物價格  $P_A$  為 2 元，而 B 物價格也為 2 元時，他以 80 元的所得購買 20 單位的 A 物，當  $P_A = 4$ ,  $P_B = 2$  時，他以 120 元的所得購買 25 單位的 A 物。請問其消費行為是否符合經濟理性？試加比較說明之。

(Definition)  $(x_1, y_1)$  is revealed preferred to  $(x_2, y_2)$

if (1) both  $(x_1, y_1)$  and  $(x_2, y_2)$  are affordable

(2)  $(x_1, y_1)$  is chosen

if  $(x_1, y_1)$  is chosen at  $(P_{x1}, P_{x2})$ ,  $P_x x_1 + P_y y_1 \geq P_x x_2 + P_y y_2$

### The principle of the revealed preference

If consumer is rational

$\Rightarrow$  Then  $(x_1, y_1)$  is revealed preferred to  $(x_2, y_2)$

implies  $(x_1, y_1)$  is preferred to  $(x_2, y_2)$

$(x_1, y_1)$  is revealed preferred to  $(x_2, y_2)$

then  $(x_2, y_2)$  cannot be revealed preferred to  $(x_1, y_1)$   
 $P_{x1}x_1 + P_{y1}y_1 \geq P_{x1}x_2 + P_{y1}y_2$  and  $(x_1, y_1)$  is chosen  
 at  $(P_{x2}, P_{y2})$ ,  $(x_2, y_2)$  is chosen, but not  $(x_1, y_1)$

$$P_{x2}x_1 + P_{y2}y_1 \geq P_{x2}x_2 + P_{y2}y_2$$

### Example

- (a)  $(P_{x1}, P_{y1}) = (20, 1)$      $(x_1, y_1) = (2, 40)$      $m_1 = 80$   
        $(P_{x2}, P_{y2}) = (20, 4)$      $(x_2, y_2) = (3, 25)$      $m_2 = 160$
- (b)  $(P_{x1}, P_{y1}) = (20, 1)$      $(x_1, y_1) = (3, 20)$      $m_1 = 80$   
        $(P_{x2}, P_{y2}) = (20, 4)$      $(x_2, y_2) = (2, 30)$      $m_2 = 160$

sol.

(a)  $P_{x1}x_1 + P_{y1}y_1 = 20 * 2 + 1 * 40 = 80$   
 $P_{x1}x_2 + P_{y1}y_2 = 20 * 3 + 1 * 25 = 85$   
 $\Rightarrow (x_1, y_1)$  is not revealed preferred to  $(x_2, y_2)$  ... (1)

$P_{x2}x_2 + P_{y2}y_2 = 20 * 3 + 4 * 25 = 160$   
 $P_{x2}x_1 + P_{y2}y_1 = 20 * 2 + 4 * 40 = 200$   
 $\Rightarrow (x_2, y_2)$  is not revealed preferred to  $(x_1, y_1)$  ... (2)  
 (1), (2)  $\Rightarrow$  doesn't violate the WARP.

sol.

(b)  $P_{x1}x_1 + P_{y1}y_1 = 20 * 3 + 1 * 20 = 80$   
 $P_{x1}x_2 + P_{y1}y_2 = 20 * 2 + 1 * 30 = 70$   
 $\Rightarrow (x_1, y_1)$  is revealed preferred to  $(x_2, y_2)$  ... (1)

$P_{x2}x_2 + P_{y2}y_2 = 20 * 2 + 4 * 30 = 160$   
 $P_{x2}x_1 + P_{y2}y_1 = 20 * 3 + 4 * 20 = 140$   
 $\Rightarrow (x_2, y_2)$  is revealed preferred to  $(x_1, y_1)$  ... (2)  
 (1), (2)  $\Rightarrow$  violate the WARP.

$$\left[ \begin{array}{l} \text{a not RP to b} \\ \text{b not RP to a} \end{array} \right] \text{ WARP ok!}$$

$$\left[ \begin{array}{l} \text{c RP to d} \\ \text{d RP to c} \end{array} \right] \text{ violate to WARP}$$

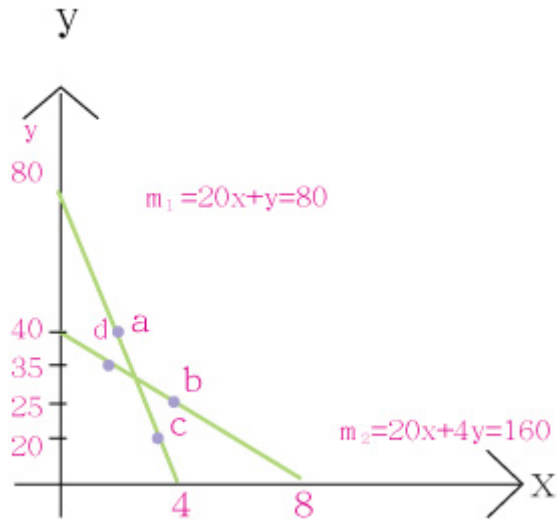


Figure 81:

at  $(P_{x1}, P_{y1})$ , e is chosen

f is affordable, e is RP to f

at  $(P_{x2}, P_{y2})$ , f is chosen

e is not affordable, f is not RP to e

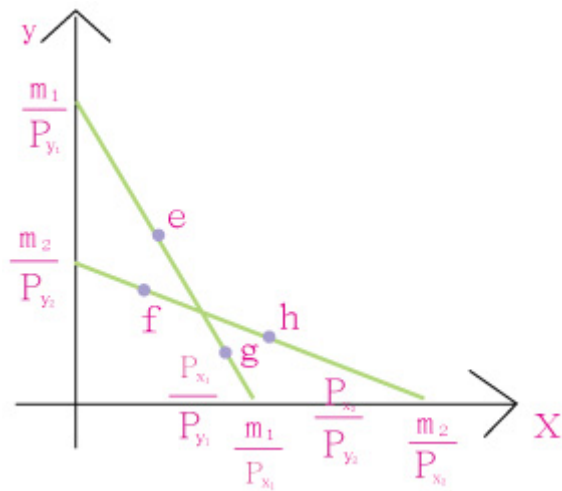


Figure 82 :

$\Rightarrow$  e, f don't violate WARP

at  $(P_{x1}, P_{y1})$ , g is chosen

h is not affordable, g is not RP to h

at  $(P_{x2}, P_{y2})$ , h is chosen

g is affordable, h is RP to g  $h > g$

$\Rightarrow$  g, h don't violate WARP

(a, b) }  
 (a, c) } ok!

(a, d) } not  
 (a, e) } ok

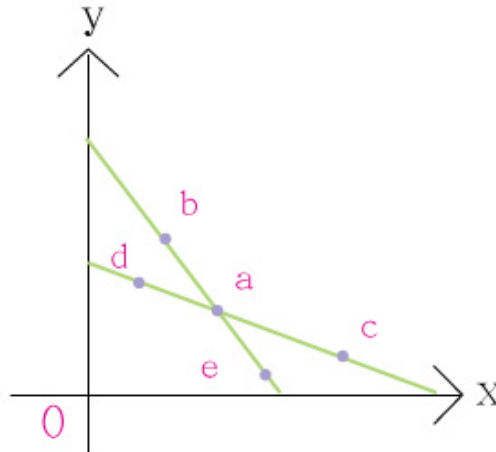


Figure83 :

Income is fixed at  $m$

at  $(P_{x1}, P_{y1})$  the consumer chooses  $a$

at  $(P_{x2}, P_{y2})$  the consumer chooses  $b$

$a \rightarrow b$  price effect of a decrease in price of  $X$

$(P_{x1} \rightarrow P_{x2}, P_{x2} < P_{x1})$

Figure89 :

How to decompose PE into SE and IE?

slutsky substitution effect.

If a consumer would have chosen  $c'$  after a Slutsky income subsidy, note that at  $P_{x1}, P_y$ , and  $m$ ,  $a$  and  $c'$  are affordable,  $a$  is chosen

$\Rightarrow a$  is RP to  $c'$

at  $P_{x2}, P_y$ , and  $m$ ,  $a$  and  $c'$  are both affordable,  $c'$  is chosen

$\Rightarrow c'$  is RP to  $a$

check  $a, c$  is OK with WARP

At  $(P_{x1}, P_y, m)$ ,  $a$  is chosen but  $c$  is not affordable

$\Rightarrow a$  is not RP to  $c$ .

At  $(P_{x2}, P_y, m)$ ,  $c$  is chosen,  $a$  and  $c$  are both affordable



$\Rightarrow c$  is RP to  $a$

$\Rightarrow$  doesn't violate WARP

$P_x \downarrow$ , Slutsky substitution effect

$$a \rightarrow c$$

$$x_3 > x_1 \text{ (X is cheaper, X substitutes for Y)}$$

After Slutsky subsidy,

$$P_{x2}x_1 + P_{y2}y_1 = P_{x2}x_3 + P_{y2}y_3 = m' \dots \Phi$$

original bundle  $a$  is  $c$  is chosen after

affordable at new price Slutsky subsidy

$\Rightarrow c$  is RP to  $a$

$a$  cannot RP to  $c \Rightarrow c$  is not affordable at  $(P_x, P_y)$

$$m = P_{x1}x_1 + P_{y1}y_1 > P_{x1}x_3 + P_{y1}y_3 \dots \Theta$$

$\Phi - \Theta$

$$(P_{x2} - P_{x1})x_1 + (P_{y2} - P_{y1})y_1 > (P_{x2} - P_{x1})x_3 + (P_{y2} - P_{y1})y_3$$

$$(P_{x2} - P_{x1})(x_3 - x_1) + (P_{y2} - P_{y1})(y_3 - y_1) < 0$$

$$P_{y2} = P_{y1} = P_y \text{ fixed}$$

$$\Rightarrow (P_{x2} - P_{x1})(x_3 - x_1) < 0$$

$$\Rightarrow P_{x2} < P_{x1} \Rightarrow x_3 - x_1 > 0$$

$$x_3 > x_1$$

### \* Tax

Tax on gasoline (X)

$\$t$  tax on each unit of X

$$P_x \rightarrow P_x + t \Rightarrow \text{equilibrium } e_1 \rightarrow e_2$$

consumer is worse off

$e_1$  is revealed preferred to  $e_2$

( $e_1$  &  $e_2$  are affordable before imposing a  $\$t$  unit tax)

$\$t^* X \Rightarrow$  a tax return to the consumer

$\Rightarrow$  new equilibrium  $e_3$

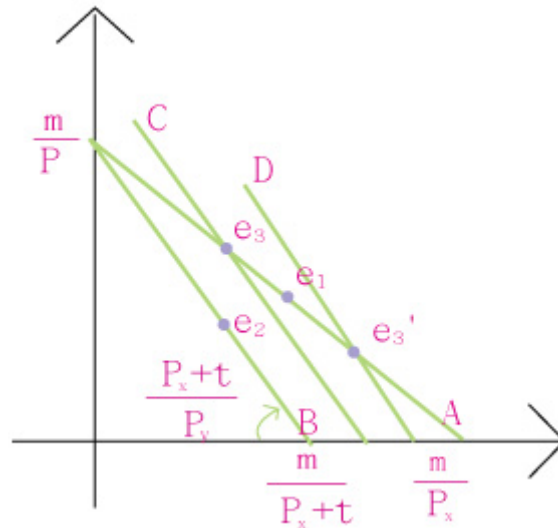


Figure 84 :

$$A: P_x x + P_y y = m$$

$$B: (P_x + t)x + P_y y = m$$

$$\text{The budget line after tax return: } (P_x + t)x + P_y y = m + tx$$

$$\Rightarrow P_x x + P_y y = m \quad \text{same as A}$$

new income:  $m' = m + tx$

new price:  $P_x + t$  &  $P_y$

slope of the budget line (after tax and tax return)

$$\Rightarrow \frac{P_x + t}{P_y} \quad (\text{a steeper budget line})$$

$$(P_x + t)x + P_y y = m + tx \quad \Rightarrow \text{both } e_1 \text{ and } e_3 \text{ are affordable at } (P_x$$

$$\Rightarrow e_1 \text{ is revealed preferred to } e_3$$

$$\Rightarrow \text{consumer is worse off at } e_3$$

Suppose new equilibrium were at  $e_3'$

the new budget line is D.

the expenditure of  $e_1$  at  $(P_x + t)$  is less than  $m'$

$\Rightarrow e_3'$  is revealed preferred to  $e_3$  (both are on A, and  $e_1$  is chosen before tax)

Based on new budget line C

$\Rightarrow e_1$  is not affordable after tax and tax return

$\Rightarrow e_3$  is not revealed preferred to  $e_1$

conclusion:  $X \downarrow$  after a tax and tax return  
 $e_3$  必在  $e_1$  左上方

### \*Price Index and welfare

based period : 0

current period : t

at period 0 :  $P_{x0}, P_{y0}$   
 $X_0, Y_0$

at period t :  $P_{xt}, P_{yt}$   
 $X_t, Y_t$

Compare welfare between periods 0 and t

the consumer is better off in period 0 (( $X_0, Y_0$ ) is RP to  $X_t, Y_t$ )

$$\text{if } P_{x0}x_0 + P_{y0}y_0 > P_{x0}x_t + P_{y0}y_t \dots(1)$$

the consumer is better off in period t (( $X_t, Y_t$ ) is RP to  $X_0, Y_0$ )

$$\text{if } P_{xt}x_t + P_{yt}y_t > P_{xt}x_0 + P_{yt}y_0 \dots(2)$$

$$m_t = P_{xt}x_t + P_{yt}y_t$$

$\frac{m_t}{m_0}$   
(1)

Paasche price index 巴氏指數

$$\frac{P_{xt}x_t + P_{yt}y_t}{P_{x0}x_0 + P_{y0}y_0} \leq \frac{P_{xt}x_t + P_{yt}y_t}{P_{x0}x_t + P_{y0}y_t} \quad \text{price index weighted by current consumption bundle}$$

$\text{Index}_p \geq \frac{m_t}{m_0} \Rightarrow$  the consumer is better off in period 0.

$\frac{m_t}{m_0}$   
(2)

Lasperes price index 拉氏指數

$$\frac{P_{xt}x_t + P_{yt}y_t}{P_{x0}x_0 + P_{y0}y_0} \geq \frac{P_{xt}x_0 + P_{yt}y_0}{P_{x0}x_0 + P_{y0}y_0} \quad \text{price index weighted by based period consumption bundle}$$

CPI is one of Lasperes price index

$\text{Index}_L \leq \frac{m_t}{m_0} \Rightarrow$  the consumer is better off in period t.